Cooperative Learning in Mathematics

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Cooperative learning is said to have developed out of studies conducted in the field of social psychology that were focused on comparing reward structures. Slavin (1980) defines cooperative learning as a generic term for “classroom techniques in which students work on learning activities in small groups and receive rewards or recognition based on their group’s performance” (Slavin, 1980, p. 315). The type of cooperative learning implemented in a classroom may vary, however, several characteristics remain constant (Kotsopoulos, 2010). Cooperative learning in a mathematics classroom involves social accountability, positive interdependence, individual accountability, and group accountability (Kotsopoulos, 2010; Leikin & Zaslavsky, 1999; Walmsley & Muniz, 2003).

**Characteristics of Cooperative Learning**

In cooperative learning, there is a level of social accountability, in which individuals are concerned for other group members’ learning (Kotsopoulos, 2010). All members should be given an equal opportunity to interact with each other and encourage the communication of mathematics ideas (Leikin & Zaslavsky, 1999). Doing this requires interpersonal skills such as communication, trust, and conflict resolution (Walmsley & Muniz, 2003). Students may need to be taught how to collaborate within their groups. For example, students can learn to value their classmate’s contributions in their groups by observing the teacher recognizing individuals for making specific contributions to the class (Jansen, 2012).

Positive interdependence refers to the idea that students are reliant on their fellow group members to learn from one another (Walmsley & Muniz, 2003; Leikin & Zaslavsky, 1999; Kotsopoulos, 2010; Jansen, 2012), that “one student’s success helps another to be successful” (Slavin, 1980, p. 316). Many of the characteristics of cooperative learning will only be present if the academic goals outlined for students require group effort (Kotsopoulos, 2010), which often depends on the task chosen, which will be further elaborated on later.

Students should also be held individually accountable, so that each member has a responsibility to contribute to the group and is accountable for the learning progress of the group (Kotsopoulos, 2010; Leikin & Zaslavsky, 1999; Walmsley & Muniz, 2003). According to Kotsopoulos (2010), in order to have individual accountability, students must be aware of the fact that what they are learning will be needed or assessed in the future. When there is a high level of individual accountability, contributions to the group are individually quantifiable (Slavin, 1980). When there is little individual accountability, or what Slavin (1980) refers to as substitutability, all group members have the same task and can substitute for one another. Without individual accountability, the group goal may still be achieved while some group members may not participate (Slavin, 1980).

Lastly, there must be group accountability or rewards (Kotsopoulos, 2010; Walmsley & Muniz, 2003). Having group rewards gives students a reason to work together. When group rewards are in place, students are more likely to care about each other’s success, listen better, and value alternative solution methods to mathematics problems (Walmsley & Muniz, 2003). When individuals are only rewarded if other members of the group succeed, high achievers are prevented from dominating the group. Students are forced to take responsibility for their learning, as well as for their classmates’ learning (Whicker, Bol, & Nunnery, 1997).

Without implementing each of these aspects, cooperative learning will not occur. Most importantly, there must be both individual and group accountability (Kotsopoulos, 2010; Walmsley & Muniz, 2003; Whicker, Bol, & Nunnery, 1997). When there are proper reward structures in place, the rest of the characteristics will fall into place.

**Reward Interdependence**

An interpersonal reward is a consequence for an individual based on the performance of classmates. When students are asked to work with one another, but there is no defined group goal, there is low reward interdependence. Students are less likely to work collaboratively if this is the case. To have high reward interdependence, there must be an explicit group reward based on group performance (Slavin, 1980).

There are three types of interpersonal reward structures: individualization, competitive, and cooperative. In individualization students’ achievement goals are unrelated. With competitive, or negative, reward interdependence, one student’s success causes another’s relative failure (Slavin, 1980). Having “one student’s success [help] another to be successful” (Slavin, 1980, p. 316), demonstrates a cooperative, or positive, reward interdependence.

Reward interdependence has an effect on group productivity. In a competitive reward structure, productivity depends on whether students feel they have a chance to succeed. When competitors are unevenly matched, there is little chance for success, so student motivation will be low; when competitors are evenly matched, students feel that their efforts may lead to success. Many teacher-centered classrooms use a competitive structure that Kagan (1989) refers to as Whole-Class Question-Answer. With this structure, the teacher asks a question; students who want to respond raise their hand, and the teacher chooses a student who then attempts to give a correct answer. In this structure, students are competing against each other, “creating poor social relations and peer norms against achievement” (Kagan, 1989, p. 13). This structure involves negative reward interdependence. For example, when a teacher asks a simple, open-ended mathematics question to the whole class, that only has one answer, only one student has a chance to respond. Often times, the student chosen to respond is the first to raise their hand, likely a high achieving student. This creates an unequal opportunity for other students, and takes away their opportunity to show what they know.

Although cooperative reward structures can reduce the productivity of an individual student, this can be offset by the increase in interpersonal rewards for individual behavior. Individual group members will be rewarded by their group for participating and blamed if they do not. Even though there may not be explicit individual rewards, students may create them within their groups (Slavin, 1980).

**Task Interdependence**

Group work was not necessarily valued when the mathematics task did not necessitate it (Jansen, 2012). When there is low task interdependence, individuals can choose to work alone without disrupting the group activity. A task that requires all group members to solve their own problems, may not lend itself to group help (Slavin, 1980). Many students define group success by the speed of completion. When this is true in a mathematics course, many low achievers feel they cannot meet their group members’ expectations (Mulryan, 1992). When students are focused on efficiency, learning opportunities may be missed (Jansen, 2012).

When there is high task interdependence, students are reliant upon each other to complete the activity (Slavin, 1980). To have high task interdependence, tasks must be challenging so that students feel the need to use each other as resources (Jansen, 2012; Kotsopoulos, 2010). Tasks chosen should have multiple entry points and solution strategies. When finding multiple solutions, students feel that groups are useful; therefore, students value the cooperative groups. Pushing for multiple solutions may lead to more collaboration because students cannot move to another questions after finding one right answer. In addition, more students have the opportunity to contribute to the group and increase their mathematics competence as perceived by themselves and their peers (Jansen, 2012).

**Benefits of Cooperative Learning in Mathematics**

There are many benefits of using cooperative learning in the classroom. Studies have shown that cooperative learning increases achievement, conceptual understanding, mathematics communication, and time spent on task. Other affective outcomes have also been observed: improved peer relations, more positive attitudes, and better self-esteem.

**Cognitive Outcomes**

According to Leikin and Zaslavsky (1999), a mathematics classroom with low student activity can negatively affect low-achieving students. In mathematics cooperative learning environments, students are more actively involved in their learning (Leikin & Zaslavsky, 1999; Mulryan, 1992) and spend more time on task (Jansen, 2012; Mulryan, 1992; Whicker et al., 1997). This increase is due mostly to the opportunity for increased student-to-student learning interactions (Leikin & Zaslavsky, 1999).

Cooperative learning has been found to produce at least the same achievement as traditional instructional methods, and most often better results (Mulryan, 1992; Slavin, 1980; Whicker et al., 1997). It has also been found specifically that competition between groups positively affects achievement (Slavin, 1980). While working in cooperative groups, students are exposed to tasks with higher cognitive demand (Mulryan, 1992; Leikin & Zaslavsky, 1997). Students demonstrated increased retention of material and improved critical thinking skills (Walmsley & Muniz, 2003).

Depending on the structure used, cooperative learning can “promote conceptual understanding of mathematics and development of mathematical reasoning skills in addition to procedural fluency” (Jansen, 2012, p. 38). When working on complex concepts or difficult problems, students found that groups made understanding and learning easier (Whicker et al., 1997). Cooperative learning helps to develop higher order thinking skills required to develop and solve problems (Noddings, 1989). Students were able to make more progress when they worked in groups than they would have made working individually (Jansen, 2012).

Cooperative learning provides more opportunities for mathematics communication (Leikin & Zaslavsky, 1997; Walmsley & Muniz, 2003). A small-group cooperative setting may promote low-level students’ mathematics communication in particular. Students are more likely to ask questions in a cooperative learning environment (Leikin & Zaslavsky, 1999), especially low achieving students (Jansen, 2012). This indicates that small-groups offer a more supportive atmosphere (Leikin & Zaslavsky, 1997) where students are able to give and receive more elaborate explanations of mathematics (Walmsley & Muniz, 2003; Leikin & Zaslavsky, 1997). Small-group communication has been found to be “effective in promoting middle-school students’ problem solving and understanding of certain mathematical structures” (Noddings, 1989, p. 619). By reasoning out loud in mathematics, students are able to make connections, develop understanding, and “fill in gaps in one’s understanding that he or she recognizes while explaining to classmates” (Jansen, 2012, p. 39). When individuals explain their reasoning to other students, they are able to refine their own thinking, while learning from one another (Kotsopoulos, 2010).

**Affective Outcomes**

While working in a cooperative group, students learn useful social and life skills. Students learn to be more tactful, manage conflict, and respect the opinions of others (Whicker et al., 1997). Students had positive attitudes towards mathematics and communicating mathematically with their peers (Leikin & Zaslavsky, 1997, 1999). They were also more motivated and enthusiastic about mathematics (Mulryan, 1992; Whicker et al., 1997). Students learn to work with others on shared problems or challenges, which is an important life skill (Kotsopoulos, 2010; Jansen, 2012).

Cooperative learning helps to improve peer relations, especially with minority students (Mulryan, 1992; Slavin, 1980). Students in cooperative groups provide each other with support and assistance (Walmsley & Muniz, 2003). Often times, students offered each other help, even when it was not requested (Leikin & Zaslavsky, 1997). This demonstrates students’ mutual concern for their group members (Slavin, 1980).

Increases in self-esteem have been observed with cooperative learning (Mulryan, 1992; Walmsley & Muniz, 2003; Whicker et al., 1997; Slavin, 1980). In mathematics, self-esteem plays a vital role in students’ motivation. Cooperative learning can reinforce self-acceptance, which makes students believe they are more academically successful, and thereby boost self-esteem (Walmsley & Muniz, 2003). Additionally, when students see their mathematics competence as malleable, they are more likely to see themselves and their peers as able contributors to the group. Promoting the belief that peers have valid ideas to share with the group also helps to improve self-esteem and increases collaboration (Jansen, 2012).

**Negative Aspects of Cooperative Learning**

When small groups do not behave cooperatively, collective goals may be met, while individual goals are not. In these groups, students do not willingly help each other and do not feel any responsibility for others’ learning. Kotsopoulos (2010) observed that group members perceived to be holding the group back, were treated in a hostile manner by other group members. Non-cooperative groups can lead to students being left behind or excluded from their groups, which can lead to feelings of incompetence (Kotsopoulos, 2010).

Students may engage in off-task behavior when working in small groups. Mulryan (1992) identified three main off task behaviors. Social opportunists were students who would talk to their peers about non-mathematics related topics. Intentional loafers would purposefully try to get others off task. These two groups consisted mostly of low achieving students. Alternative involvement, meaning students were working on other school assignments, was observed for both high and low achieving students (Mulryan, 1992).

**Student Passivity**

Theoretically, cooperative learning should allow “for more active involvement in mathematical learning tasks and more opportunities for collaboration with peers on these tasks than conventional instructional practices, [however] students who fail to take advantage of these opportunities may have little to gain from cooperative small-group work” (Mulryan, 1992, p. 266). Passivity is defined as the failure or unwillingness to participate in the group activity or work with other group members. A passive student is one who does not ask questions, offer explanations, or respond to other students’ questions. Students either withdraw from the activity or depend on others to complete the assignment. When this is the case, “passive students may suffer both academically and emotionally” (Mulryan, 1992, p. 266).

Mulryan (1992) outlines six categories of passive students: discouraged, unrecognized, despondent, unmotivated, bored, or intellectual snob. Students may be discouraged by the difficulty of the task and decide to leave it to those who understand. Some students offer to help, but are ignored or unrecognized by the group. Despondent students disliked their group. If students feel that the task is unimportant they fall into the category of unmotivated. Students who were uninterested or bored with the task, often because the task was too easy, became disengaged. Students who felt that their peers were less competent in mathematics and didn’t feel like having to explain their reasoning were considered intellectual snobs. These students would often choose to work on their own. Mostly low achievers fell into the categories of discouraged or unrecognized. Similar numbers of high and low achievers were grouped as despondent or unmotivated. Mostly high achievers were classified as bored or intellectual snobs (Mulryan, 1992).

High achievers were more likely to either dominate the group or work alone. These students were more likely to be on task, be actively participating, and collaborating with group members. High achieving students provided the group with more suggestions and directions (Mulryan, 1992). Low achievers were more likely to be passive than high achievers in cooperative settings (Jansen, 2012; Mulryan, 1992; Whicker et al., 1997). This is similar to observations of whole-class settings. Low achievers are known to become more and more passive in the classroom throughout their educational careers (Mulryan, 1992).

Low achievers may be more passive because of the way they are treated by their high achieving group members. Some high achievers indicated that they disliked helping low achievers and would avoid helping them. Many low achievers are likely to have experienced more failures in mathematics, which creates a negative self-efficacy that carries over into their small groups (Mulryan, 1992).

**Improving Negative Outcomes**

In order to decrease the negative effects of cooperative learning, teachers can increase individual accountability, use self-surveillance, and promote social transparency. By increasing individual accountability, students will be more likely to complete the group task (Mulryan, 1992). Kotsopoulos (2010) feels that students need to be aware of their actions and know that others are aware of their actions. This awareness should promote a collective effort to value others. This may be a starting point for allowing students that are excluded to become included members (Kotsopoulos, 2010). In a study conducted by Kotsopoulos (2010), she used a self-surveillance method, where students watched videos of their groups and reflected on what they saw. This allowed excluded students to question each others’ group participation. When students are able to question each other within the group, power is more evenly distributed. After watching the videos, students changed their behaviors. Self-surveillance has the potential to “disrupt non-collaborative learning by allowing students to question their roles in others’ learning” (Kotsopoulos, 2010, p. 138).

**Cooperative Learning Methods**

Cooperative activities are lessons that are specific to a certain subject area or topic. Cooperative structures, however, are cooperative learning instructional methods that can be used in any subject, with a variety of ages, and throughout various portions of a lesson. There are numerous cooperative structures (Kagan, 1989), however, we will focus on the Jigsaw method, Teams-Games-Tournaments (TGT), and Student Team-Achievement Divisions (STAD), since each of these structures has been studied in mathematics courses.

**Jigsaw**

In Aronson’s Jigsaw method, students are placed in small heterogeneous groups (Sharan, 1980; Slavin, 1980). The task is divided into pieces (based on the number of students in the groups), which are assigned to group members based on ability level (Sharan, 1980; Slavin, 1980). Groups split up and meet with individuals from other groups who were given the same portion of the task. These groups then work together to become experts on the material. After students have become experts on their portion of the task, they return to their original groups and teach the material to their jigsaw group members (Kagan, 1989; Sharan, 1980; Slavin, 1980). By the end of the lesson, students are expected to know all the pieces of the lesson and are assessed individually (Kagan, 1989; Slavin, 1980).

This method involves a high level of positive interdependence to complete the task. Students cannot learn all of the material without their group members (Kagan, 1989; Sharan, 1980). Although individual outcome is dependent on team members’ performance, there is no formal group goal, since quiz scores affect individuals only. According to Slavin (1980), this low reward interdependence is counteracted by the high task interdependence, which indirectly creates reward interdependence.

In Slavin’s (1980) review of cooperative learning research, he concluded that the results of studies involving the Jigsaw method were not very clear on its effect on race relations, achievement, liking of school, and mutual concern. Sharan (1980) however, found significant achievement gains for minority groups. One consistent result found when using the Jigsaw method was an increase in self-esteem (Sharan, 1980; Slavin, 1980). This is likely due to the fact that each student serves as a resource for the group. It was also found to increase students’ ability to see other perspectives and to increase internal locus of control (Slavin, 1980).

**Teams-Games-Tournament**

DeVries’ Teams-Games-Tournaments (TGT) consists of 4 to 5 member heterogeneous teams. Team members prepare each other to do well in the tournament to be conducted the next day over the material taught by the teacher. Tournament tables are grouped by ability level, so that competition is fair. Students compete as a representative of their team on the material presented by the teacher and on the study sheet. Individual’s scores are added to their team scores (Sharan, 1980; Slavin, 1980). Team assignments remain the same, but tournament tables may vary from week to week based on the student’s performance (Slavin, 1980).

By having tournament tables organized by ability level, all students are given a chance to succeed (Sharan, 1980). Because team scores are figured by adding up individual team members’ scores, positive reward interdependence is created (Sharan, 1980). All team members are able to contribute to the group.

Studies have shown that TGT has positive effects on achievement, race relations, and mutual concern. An increased liking of school was also observed. Students felt that their peers supported and encouraged their academic success (Slavin, 1980). Based on a study reviewed in Slavin’s “Cooperative Learning” (1980), it may be that in TGT, the opportunity for student interaction and help, do not affect performance, but team recognition and peer norms favoring achievement from cooperative reward structures do. Similar results were found in a study using the STAD structure (Slavin, 1980).

**Student Teams-Achievement Divisions**

Student Teams-Achievement Divisions (STAD), created by Robert Slavin is conducted in a similar manner to TGT (Sharan, 1980). Students are grouped in small heterogeneous teams (Sharan, 1980; Slavin, 1980; Whicker et al., 1997). Students are to explain their answers to each other and ask questions of teammates rather than the teacher (Whicker et al., 1997). After studying with their teams, instead of competing in a tournament, students take a quiz. The quiz scores are then converted to team scores, based on performance within an academic division. These divisions consist of students with similar ability levels, ensuring that students have an equal chance of earning points for their group. Students are not aware of who else is in their division. Academic divisions change from week to week, based on previous performance (Sharan, 1980; Slavin, 1980).

In order for the group to do well, all members must learn the material and receive points (Whicker et al., 1997), thus there is high reward interdependence (Slavin, 1980). Students had more positive attitudes and were more likely to help each other than students in a traditional classroom. Students taught using STAD expressed a mutual concern for peers (Sharan, 1980; Slavin, 1980), had more positive race relations, showed increased learning, and more on-task behavior (Slavin, 1980).

**Selecting a Cooperative Structure**

“Implementing any cooperative small-group setting does not automatically ensure cooperative work and a positive effect on all students” (Leikin & Zaslavsky, 1997, pp. 333-334).Different structures are based on different educational philosophies that “lead to variations in types of learning and cooperation, student roles and communication patterns, teacher roles, and evaluation” (Kagan, 1989, p. 12). Structures serve a variety of functions and uses. Students’ experiences and interactions within their groups depend on the learning objectives and the teacher’s expectations (Jansen, 2012; Leikin & Zaslavsky, 1997). It is necessary for the teacher to decide whether mastery or content development is expected (Kagan, 1989) and the type of grouping that will be used.

**Learning Objectives**

One way to use cooperative groups is to help improve basic mathematics skills (Noddings, 1989). TGT and STAD focus heavily on computation and procedural learning (Slavin, 1980; Noddings, 1989). These methods also employ competition between groups, which promotes accuracy and speed. Therefore, these methods may not be useful when the teacher feels that mathematical intelligence is socially constructed (Noddings, 1989). To develop high levels of cognitive learning outcomes, a less structured cooperative learning setting should be used. Cooperative structures that are less structured allow for greater student autonomy, which facilitates deeper understanding (Slavin, 1980).

**Grouping**

When students are placed in homogeneous groups by ability level, the teacher’s role does not necessarily change significantly (Noddings, 1989). In this case, small groups are used to enhance the teacher’s instruction through differentiation. The teacher can ensure that students receive proper instruction by adapting the pacing or giving different homework and tests (Noddings, 1989).

Student-centered learning is said to induce learning and communication through peer interaction. Some teachers and researchers advocate that the stimulation provided by small groups helps produce higher quality mathematics learning than direct instruction. Student-centered classrooms usually emphasize group processes, problem solving, attitudes, and social development (Noddings, 1989). These benefits may take time to see, as bonds between group members take time to form. As students progress, they become decreasingly dependent on their teacher (Whicker et al., 1997). According to Noddings (1989), students in small groups are able to learn as well by themselves as they are with the teacher.

When students are grouped heterogeneously, the teacher is no longer the primary source of information (Sharan, 1980). Leiken and Zaslavsky (1999) claim that students learn mathematics better in groups with a mixture of ability levels. Mulryan (1992) and Noddings (1989), however, found that while high achieving and low achieving students do better in heterogeneous groups, middle achievers do better in homogeneous groups. Low-achieving students benefit from being able to ask questions (Noddings, 1989). Teachers that are focused on “engagement, problem seeking and formulation, multiple solutions,” (Noddings, 1989, pp. 614-615) and other similar ideas would likely use heterogeneous small-groups (Noddings, 1989).

**Conclusion**

There are many cooperative learning structures that can be used in a mathematics classroom, three of which are TGT, STAD, and Jigsaw. Despite the variety of these structures, social accountability, positive interdependence, individual accountability, and group accountability are always present. Reward and task interdependence affect the types and levels of cooperation that will occur in small-groups. When there is high reward and task interdependence, students may benefit in several ways. Students spend more time on-task and are more active in their learning. Achievement, mathematics communication, and conceptual understanding can be increased. Students develop a more positive attitude towards mathematics, have improved peer relations, and increased self-esteem. As in any classroom setting, there are negative aspects of cooperative learning, the most prevalent being student passivity. With all of these aspects of cooperative learning, teachers must be aware that their expectations influence student outcomes. Thus, when choosing a cooperative learning structure to use, it is important for teachers to consider the goals they want their students to achieve. These goals will determine the cooperative structure to be implemented for optimum student learning.

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